## Subordination algebras and closed relations between compact Hausdorff spaces

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De Vries duality [3] establishes a dual equivalence between the category KHaus of compact Hausdorff spaces and continuous functions and the category DeV of de Vries algebras and de Vries morphisms. To each compact Hausdorff space X it is associated the de Vries algebra  $\mathcal{RO}(X)$  of its regular open subsets equipped with the relation  $\prec$  defined by  $U \prec V$  iff  $cl(U) \subseteq V$ . In this talk we see how to lift de Vries duality to the category KHaus<sup>R</sup> of compact Hausdorff spaces and closed relations between them, where a binary relation  $R: X \to Y$  is said to be *closed* if it is a closed subset of  $X \times Y$ .

We show that  $\mathsf{KHaus}^\mathsf{R}$  is equivalent to the category  $\mathsf{SubS5}^\mathsf{S}$  of S5-subordination algebras and compatible subordinations between them. S5-subordination algebras are boolean algebras equipped with binary relations that generalize the relations on de Vries algebras. In addition, this equivalence restricts to an equivalence between  $\mathsf{KHaus}^\mathsf{R}$  and the subcategory  $\mathsf{DeV}^\mathsf{S}$  of  $\mathsf{SubS5}^\mathsf{S}$  whose objects are de Vries algebras. This yields a generalization of de Vries duality to closed relations between compact Hausdorff spaces. We also describe the subcategories of  $\mathsf{SubS5}^\mathsf{S}$  and  $\mathsf{DeV}^\mathsf{S}$  that are equivalent to  $\mathsf{KHaus}$  and mention their advantages over  $\mathsf{DeV}$ .

The last part of the talk is about MacNeille and ideal completions of S5-subordination algebras, which yield direct choice-free equivalences and dual equivalences between  $SubS5^{S}$ ,  $DeV^{S}$ , and the category of compact regular frames and preframe homomorphisms.

This talk is based on the joint works [1, 2] with M. Abbadini and G. Bezhanishvili.

## References

- [1] M. Abbadini, G. Bezhanishvili, and L. Carai, A generalization of de Vries duality to closed relations between compact Hausdorff spaces, 2022, arXiv:2206.05711.
- [2] \_\_\_\_\_, Ideal and MacNeille completions of subordination algebras, 2022, arXiv:2211.02974.
- [3] H. de Vries, *Compact spaces and compactifications. An algebraic approach*, Ph.D. thesis, University of Amsterdam, 1962.