Algebraic Semantics of n-valued Modal Logics

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In the present work, we focus our attention on a many-valued modal system based on the n-valued Łukasiewicz logic $\Lambda(\mathbf{Ł}_n)$ (for each $n \in \mathbb{N}$). Our idea is to extend $\Lambda(\mathbf{Ł}_n)$ to a modal system, by adding a unary operator. To that aim, we recall that the equivalent algebraic semantics of $\Lambda(\mathbf{Ł}_n)$ is the subvariety of MV-algebras generated by the MV-chain with n elements $\mathbf{Ł}_n$. Our algebraic approach is done by considering complex algebras that arise from $Ł_n$-valued Kripke frames, that is, frames such that the accessibility function takes values in the chain $Ł_n$ and the models are also evaluated in $\mathbf{Ł}_n$. We define the quasivariety of algebras generated by these complex algebras, and this quasivariety, together with the abstract theory of algebraizable logics immediately provide an axiomatization for the minimal many-valued system over $Ł_n$. From the way that the system is defined, it turns out to be complete with respect to the logic semantically defined by the $Ł_n$-valued Kripke frames. So the logical system determined by frames over $Ł_n$ has an algebraic semantics based on MV-algebras.

We extend some of the ideas for the logic semantically defined by $Ł_n$-valued possibilistic frames. Our investigation provides a negative answer to a conjecture of P. Hájek posed in his book [4] which intends to generalize the classical setting, where the possibilistic logic coincides with the modal logic $KD45$. We prove that the logic semantically defined by $Ł_n$-valued possibilistic frames, can not be axiomatized by simply requiring the fuzzy analogues of the classical axioms K,D,4 and 5.

The ideas of the talk are based on the papers [1], [2] and [3], which are joint works with P. Cordero, M. Marcos and R. Rodríguez.

References

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[2] M. Busaniche, P. Cordero and R. Rodriguez, Corrigendum to "Algebraic semantics for the minimum many-valued modal logic

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[3] M. Busaniche, P. Cordero, M. Marcos and R. Rodriguez, "An algebraic semantics for possibilistic finite-valued Łukasiewicz logic", draft.

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