On subresiduated lattices and some generalizations

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Subresiduated lattices (sr-lattices for short) were introduced during the decade of 1970 by Epstein and Horn [1] as an algebraic counterpart of some logics with strong implication previously studied by Lewy and Hacking. A sr-lattice is a pair (A, D), where A is a bounded distributive lattice, D is a bounded sublattice of A and for every $a, b \in A$ there exists the maximum of the set $\{d \in D : a \wedge d \leq b\}$, which will be denoted by $a \to b$. In particular, $D = \{a \in A : 1 \to a = a\}$. The pair (A, D) can be regarded as an algebra $(A, \wedge, \vee, \to, 0, 1)$ of type $(2, 2, 2, 0, 0)$. Moreover, the class of sr-lattices is a variety. This variety properly contains the variety of Heyting algebras and it is also properly contained in the variety of weak Heyting algebras, which was introduced and studied by Celani and Jansana in [4].

In this talk we study some subreducts of sr-lattices [2]. We also consider a variety whose members, introduced in [5] under the name of srl-monoids, generalize sr-lattices. In particular, we study some subreducts of integral srl-monoids following some ideas and techniques developed in [2, 3].

References

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