Non-Classical Logic Days

11&14 of October of 2022

Tuesday 11

09.00	Welcoming Reception
09.15	*Finitely presented algebras in subvarieties of BL-algebras and geometric applications* **Noemi Lubomirsky**
10.00	Coffee Break
10.30	*Relevant reasoners in a classical world* ***Igor Sedlar***
11.15	*The bi-intuitionistic logic of co-trees* **Miguel Martins**
12.00	Coffee Break

Friday 14

10.00	*SAT and MaxSAT problems in Łukasiewicz logic* **Zuzana Haniková**
10.45	*Łukasiewicz logic reasons about probability* **Sara Ugolini**
11.30	One-variable lattice-valued logics **Petr Cintula**
12.15	Coffee Break

Venue: Institut de Matemàtica, Universitat de Barcelona
Gran via de les Corts Catalanes, 585, 08007, Barcelona (this is a Google Maps link)

Abstracts

Finitely Presented Algebras in Subvarieties of BL-algebras and Geometric Applications

11 Oct
09:15

Noemi Lubomirsky

CMaLP, Universidad Nacional de La Plata, Argentina
CONICET

In this talk we will give a description of the finitely presented algebras for some subvarieties of BL-algebras. In the case of the subvariety of MV-algebras, there is a correspondence between finitely presented algebras and rational polyhedra, and Mundici proved that there is a bijective correspondence between weighted abstract simplicial complexes and equivalence classes of finitely axiomatizable theories. Our aim is to extend some of these results and show the conditions that are needed to generalize that notion to varieties of BL-algebras which properly contain the variety of MV-algebras.

Relevant Reasoners in a Classical World

11 Oct
10:30

Igor Sedlar

Institute of Computer Science of the Czech Academy of Sciences, Prague, Czech Republic

We outline a framework for epistemic logic which combines classical propositional logic with substructural modal logic. In particular, the propositional fragment of our logics is classical, while epistemic attitudes of agents are assumed to be closed only under certain implications valid in specific substructural modal logics, not under arbitrary implications valid in the epistemic logic itself. Intuitively, this captures the idea that agents are assumed to infer only the ”relevant” consequences of the information at their disposal. The main technical result is a general completeness theorem. We also outline some topics of ongoing and future work.

---

11 Oct
11:15

The Bi-Intuitionistic Logic of Co-trees

Nick Bezhanishvili[1], *Miguel Martins[2] and Tommaso Moraschini[3]

[1] *University of Amsterdam, Amsterdam, The Netherlands*
[2] *Universitat de Barcelona, Barcelona, Spain*
[3]Departament de Filosofia, Facultat de Filosofia, Universitat de Barcelona (UB), Carrer Montalegre, 6, 08001 Barcelona, Spain

A bi-Heyting algebra validates the Gödel-Dummett axiom $(p \to q) \vee (q \to p)$ iff the poset of its prime filters is a disjoint union of co-trees (i.e., order duals of trees). Bi-Heyting algebras of this form are called bi-Gödel algebras and form a variety that algebraizes the extension bi-LC of bi-intuitionistic logic axiomatized by the Gödel-Dummett axiom. In this paper we initiate the study of the lattice Λ(bi-LC) of consistent extensions of bi-LC. We develop the method of Jankov formulas for bi-Gödel algebras and use them to prove that Λ(bi-LC) has the size of the continuum. Furthermore, we introduce a sequence of co-trees, called the finite combs, and show that a logic in Λ(bi-LC) is locally tabular iff it contains the Jankov formula associated with some finite comb. It follows that there exists a unique pre-locally tabular extension of bi-LC. These results contrast with the case of the intermediate logic axiomatized by the Gödel- Dummett axiom, which is known to have countably many extensions, all of which are locally tabular.

---

14 Oct
10:00

SAT and MaxSAT Problems in Łukasiewicz Logic

Zuzana Haniková

Institute of Computer Science of the Czech Academy of Sciences, Prague, Czech Republic

I will report on a work in progress. Satisfiability problems pertain to a semantics of a logic. Considering the algebraic semantics, SAT can be viewed as a relation between formulas on the one hand and algebras (matrices) on the other, in a fixed language. For the language of Łukasiewicz logic, often the “standard” MV-algebra on the real interval [0,1] is considered as semantics, whereupon the SAT problem, more or less explicitly, pertains to this algebra. Thanks to a series of results starting with a key paper of Mundici from 1987 and concluding with Aguzzoli’s results from 2006, we know that if a formula is satisfiable in the standard MV-algebra, then it is so already in its finite subalgebra, with a tight upper bound on the cardinality of such subalgebra. This talk will be about extending these results to a variant of the maximum satisfiability (MaxSAT) problem as already considered — for the three-element MV-algebra — by Li, Manya, and Vidal in 2020; namely, given an ordered n-tuple of (arbitrary) formulas in the language of Łukasiewicz logic, which is the maximum number of them that can be satisfied by a single assignment, over all assignments.

Łukasiewicz Logic Reasons About Probability

14 Oct
10:45

Sara Ugolini

Artificial Intelligence Institute, Spanish National Research Council

The interconnection between logic, algebra, and probability has played a central role in the study of reasoning since the dawn of modern logic, particularly in the groundbreaking work of Boole. More recent times have seen a flourishing of formal methods and logical approaches to deal with logics capable of reasoning with probabilities.

In joint work with Flaminio, we are concerned with the logic FP(L,L), that has been recently proved to be the logic of state theory: a generalization of probability theory for uncertain quantification on Łukasiewicz sentences. In FP(L,L), Łukasiewicz logic plays a twofold role: it is the inner logic that represents the formulas that fall under the scope of the modality P (i.e., events) and it is also the outer logic that reasons on complex probabilistic modal formulas.

We show that, roughly speaking, the modal expansion leading to the logic FP(L,L) is not needed to formalize probabilistic reasoning within Łukasiewicz calculus. Phrased in the algebraic setting, we show that the quasi-equational theory of MV-algebras is expressive enough to encode probabilistic reasoning.

One-Variable Lattice-Valued Logics

14 Oct
11:30

*Petr Cintula[1], George Metcalfe[2] and Naomi Tokuda[3]

[1]*Institute of Computer Science of the Czech Academy of Sciences, Prague, Czech Republic*

[2,3]*Mathematical Institute, University of Bern, Switzerland*

The one-variable fragment of any first-order logic may be considered as a modal logic, where the universal and existential quantifiers are replaced by a box and diamond modality, respectively. In several cases, axiomatizations of algebraic semantics for these logics have been obtained: most notably, for the modal counterparts S5 and MIPC of the one-variable fragments of first-order classical logic and intuitionistic logic, respectively. Outside the setting of first-order intermediate logics, however, a general approach is lacking.

This paper provides the basis for such an approach in the setting of first-order lattice-valued logics, where formulas are interpreted in algebraic structures with a lattice reduct. In particular, axiomatizations are obtained for modal counterparts of one-variable fragments of a broad family of these logics by generalizing a functional representation theorem of Bezhanishvili and Harding for monadic Heyting algebras. An alternative proof-theoretic proof is also provided for one-variable fragments of first-order substructural logics that have a cut-free sequent calculus and admit a certain bounded interpolation.
