A Categorical Equivalence for Odd or Even Involutive FL_e-chains

Sándor Jenei[1]

Eszterházy Károly Catholic University, Eger and University of Pécs, Pécs, Hungary
jenei.sandor@uni-eszterhazy.hu, jenei@ttk.pte.hu

The main objective of this talk is to lift the established one-to-one correspondence, as introduced in [6], between the class of even or odd involutive FL_e-chains and the class of bunches of layer groups, to a categorical equivalence [4]. In [6] a novel decomposition method, called *layer algebra decomposition*, which seems to be original not only in the field of residuated lattices but also in algebra at large (see footnote 4 in [6]), along with the corresponding construction method, have been introduced for the class of odd or even involutive FL_e-chains. The main idea was to decompose the algebra with the help of its local unit function $x \mapsto x \to x$ into a direct system, indexed by the positive idempotent elements of the algebra, of (hopefully simpler, "nicer") algebras, with transitions of the direct system defined by multiplication by a positive idempotent element. The decomposed algebra could be reconstructed through a combination of Płonka sums, as introduced by Płonka [9], and the concept of directed lexicographic order, introduced in [6] (see Remark 4.3). The impact of the layer algebra decomposition soon extended beyond its initial application and has been employed to structurally describe various classes of residuated lattices. These include finite commutative, idempotent, and involutive residuated lattices [8], finite involutive po-semilattices [7], and locally integral involutive po-monoids and semirings [1, 2]. In these classes layer algebras are "nice". However, in [6] the obtained layer algebras are only somewhat nicer than the original algebra, therefore a second phase, involving the construction of layer groups from layer algebras, was introduced. The combination of the layer algebra decomposition and this additional phase establishes a one-to-one correspondence between the class of even or odd involutive FL_e-chains and the class of bunches of layer groups. With the obvious choice for morphisms between FL_e-chains, our primary focus in this talk is to determine the appropriate notion of morphisms for the class of bunches of layer groups, and to present a functor. Due to space constraints here, we direct the interested reader to [4, Definition 2.3 and Remark 2.6] for the description of bunches of layer groups (the objects of the category of bunches of layer groups), to [4, Definition 3.3] for the description of bunch homomorphisms (the morphisms of the category of bunches of layer groups), and to [4, Theorem 3.6] for an explanation of a functor mapping to the category of even or odd involutive FL_e-chains. As a forward-looking note, it's worth noting that the categorical equivalence presented in this talk (and in [4]), has proven to be a potent tool for establishing amalgamation and densification results in classes of involutive FL_e-algebras that are neither integral, nor divisible, nor idempotent [3, 5].

References

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