# Being, Becoming, and the dimension of combinatorial spaces

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Lawvere's 1990 Thoughts of the Future of Category Theory [2] outlines a positive mathematical programe; an "attempt, by an admirer of rational mechanics, to include objective logic among the tools for arriving at a more accurate conception of space". We concentrate here on two of the philosophical guides there, and a conjecture relating them.

Partially motivated by the opposition between 'gros' and 'petit' in Algebraic Geometry, the first philosophical guide is that there is a distinction between a *general category of Being* and *particular categories of Becoming*. The terminology reflects the "hope that sober application, of category theory to the ancient philosophical categories, will not only clarify both but also renew respect for serious thought, through solid examples approaching adequacy to their concept." Alternatively, we may phrase the distinction as one between toposes 'of spaces' [3] on the one hand and 'generalized locales' on the other. Additionally, the philosophical guide includes a way to relate the two classes of toposes: each space $X$ (i.e. an object in a category of spaces) should determine a generalized locale $\mathrm{P}(X)$ of 'pseudo-classical sheaves' on $X$.

The second philosophical guide is that each topos of spaces determines a poset of dimensions which may be identified with the poset of *levels* (i.e. essential subtoposes) of the given topos.

The conjecture relating the two guides is that the dimension of a space $X$ depends only on the generalized locale $\mathrm{P}(X)$. "In other words, if we have an equivalence of categories $\mathrm{P}(X) \cong \mathrm{P}(Y)$, then $X, Y$ should belong to the same class of UIO levels within the category of Being in which they are objects. Suitable hypotheses to make this conjecture true should begin to clarify the relationships between the two suggested philosophical guides."

We show that the conjecture holds for a solid class of examples of presheaf toposes including that of simplicial sets. Moreover, we prove that the way in which $\mathrm{P}(X)$ determines the dimension of $X$ is almost identical to that in which the algebra of open subpolyhedra of a polyhedron determines its dimension [1].

## References


[1] N. Bezhanishvili, V. Marra, D. McNeill, and A. Pedrini. Tarski's theorem on intuitionistic logic, for polyhedra. *Ann. Pure Appl. Logic*, 169(5), 2018.

[2] F. W. Lawvere. Some thoughts on the future of category theory. Lect. Notes Math. 1488, 1-13, 1991.

[3] F. W. Lawvere. Alexander Grothendieck and the modern conception of Space. Invited talk at CT2015, Aveiro, Portugal, 2015.