Game comonads and resource-sensitive model theory

Luca Reggio

University College London, UK
l.reggio@ucl.ac.uk

Since the pioneering work of Lawvere in the 1960s, category theory has been used to provide a syntax-independent view of the fundamental structures of logic, including e.g. first-order logic and extensions to infinitary and higher-order languages.

In this talk, motivated by the needs of finite model theory and descriptive complexity, I will focus on logic fragments that involve a finite amount of *logical resources*, such as finite-variable logics or logics with bounded quantifier rank, and the corresponding combinatorial parameters of (relational) structures. A key insight due to Abramsky, Dawar and their collaborators is that, in many cases of interest, these resource-sensitive logics can be described by comonads on the category of structures, and the associated combinatorial parameters by the coalgebras for the comonads. This is at the origin of the framework of *game comonads* [1, 5].

I will survey the main ideas underlying game comonads and the axiomatic approach of *arboreal categories* [2, 3], and some of their applications. The latter are due to several teams of authors and include:

(i) A categorical view of homomorphism counting results in finite model theory [6, 7].

(ii) The axiomatic study of homomorphism preservation theorems in logic [4].

(iii) The interplay between Gabriel–Ulmer duality and the expressive power of arboreal categories [9].

(iv) A homotopical view of modal logic and the Łoś–Tarski preservation theorem [8].

References

[1] Abramsky, S.; Dawar, A.; Wang, P. The pebbling comonad in finite model theory. *32nd Annual ACM/IEEE Symposium on Logic in Computer Science (LiCS)*, 2017, 1–12.

[2] Abramsky, S.; Reggio, L. Arboreal categories and resources. *48th International Colloquium on Automata, Languages, and Programming (ICALP)*, 2021, 115:1–115:20.

[3] Abramsky, S.; Reggio, L. Arboreal categories: An axiomatic theory of resources. *Logical Methods in Computer Science*, 2023, 14:1–14:36.

[4] Abramsky, S.; Reggio, L. Arboreal categories and equi-resource homomorphism preservation theorems. *Annals of Pure and Applied Logic*, 2024, 103423.

[5] Abramsky, S.; Shah, N. Relating structure and power: Comonadic semantics for computational resources. *27th EACSL Annual Conference on Computer Science Logic (CSL)*, 2018, 2:1–2:17.

[6] Dawar A.; Jakl, T; Reggio, L. Lovász-type theorems and game comonads. *36th Annual ACM/IEEE Symposium on Logic in Computer Science (LiCS)*, 2021.

[7] Reggio, L. Polyadic sets and homomorphism counting. *Advances in Mathematics*, 2022, 108712.

[8] Reggio, L. A model category for modal logic. https://arxiv.org/abs/2310.12068.

[9] Reggio, L.; Riba, C. Finitely accessible arboreal adjunctions and Hintikka formulae. https://arxiv.org/abs/2304.12709.